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RECTANGULAR-WIND-TUNNEL BLOCKING CORRECTIONS USING THE VELOCITY-RATIO METHOD

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SUMMARY

In this report calculations of the ratios of the velocity increments at test bodies to those at the tunnel walls caused by the solid blocking of these bodies within the walls of a closed rectangular wind tunnel are presented. The bodies treated include two-dimensional airfoils; small bodies of revolution; straight, untapered, finite-span wings of varying span; and swept, untapered, finite-span wings of varying span. It is shown that, after wake blocking effects have been removed, the present method furnishes semiempirical blocking corrections for most wind-tunnel models and their components. Results are presented for all the cases mentioned. The test-section proportions of the Southern California Cooperative Wind Tunnel at the California Institute of Technology (viz., ratio of height to width equal to $1/\sqrt{2}$) are used in calculations.

INTRODUCTION

The velocity-ratio method of obtaining blocking corrections in high-speed, subsonic wind tunnels was first solved by Göthert for the cases of a body of revolution and of a finite-span wing of span-to-diameter ratios of 0.25 and 0.5 in a closed circular tunnel (reference 1). This work was later extended in an unpublished report to the cases of a wing having a span-to-diameter ratio of 0.75 and of a wing spanning a closed circular wind tunnel.

In the present report, the methods of reference 2 are used to extend the previous results for straight wings of varying span in a closed rectangular tunnel whose height-to-width ratio is $1/\sqrt{2}$. For swept wings, a slightly different approach involving the use of line doublets has been utilized.

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SYMBOLS

a distance of point source and sink from doublet origin

A center, or axis of test section

c wing chord

d maximum diameter of a body of revolution

g x-coordinate of a source segment

H tunnel height

l length of a body of revolution

M Mach number, corrected for blocking

M_u tunnel calibration Mach number, uncorrected for blocking (may include strut calibration)

m integer, indicating image number in y-direction

n integer, indicating image number in z-direction

q line-source strength, square feet per second

q' point-source strength, cubic feet per second

r distance from source or doublet element to point at which velocity increment shall be obtained; $r^2 = x^2 + y^2 + z^2$

R remainder

s semispan of model wing (measured in y-direction)

$s' = s/W$

t maximum wing thickness

u total axial velocity increment in test section due to all images except primary one (model)

u' total axial velocity increment in test section due to all images, including primary one (model)

u_1 axial velocity increment in test section due to a single image

U tunnel axial velocity

v_x velocity in x-direction due to a point doublet

W tunnel width

w radial coordinate; $w^2 = y^2 + z^2$

x axial coordinate

$x_1 = x$

y lateral coordinate

$y_1 = y - mW$

z vertical coordinate

$z_1 = z - nH$

$\theta = \sin^{-1} \frac{g}{r}$

μ line-doublet strength, cubic feet per second

μ' point-doublet strength, feet^4 per second

Λ angle of sweep of a given wing at any Mach number M_u

Λ_0 equivalent angle of sweep at $M_u = 0$

ψ Stokes stream function, cubic feet per second

Subscripts:

A, B, C, and so forth — particular points in test section or on test-section walls

For further explanation of the symbols see figures 1 to 4.

DETERMINATION OF INTERFERENCE VELOCITIES

Two-Dimensional Wing

A two-dimensional wing may be represented by a chordwise distribution of infinite line sources and sinks. The axial velocity increment produced by any single infinite-line-source image (fig. 1) is

$$u_1 = \frac{q}{2\pi r}\sin\theta = \frac{qg}{2\pi r^2} \tag{1}$$

at a point A in the center of the basic tunnel, $r_A{}^2 = n^2H^2 + g^2$. Thus,

$$u_{1_A} = \frac{qg}{2\pi}\left(\frac{1}{n^2H^2 + g^2}\right) = \frac{qg}{2\pi H^2}\left(\frac{1}{n^2 + \frac{g^2}{H^2}}\right) \tag{2}$$

Omitting the central source and summing for the remainder gives

$$u_A = \frac{qg}{\pi H^2}\sum_{n=1}^{\infty}\left(\frac{1}{n^2 + \frac{g^2}{H^2}}\right) \tag{3}$$

Similarly, the axial velocity at the wall of the tunnel, point B, including both the central source and all the images, is

$$u'_B = \frac{qg}{\pi H^2}\sum_{n=1}^{\infty}\left[\frac{1}{\left(n - \frac{1}{2}\right)^2 + \frac{g^2}{H^2}}\right] \tag{4}$$

For normal chord sizes, g is small compared with the tunnel dimensions and hence g^2/H^2 may be neglected in comparison with n^2 or $\left(n - \frac{1}{2}\right)^2$, with the result

$$\frac{u_A}{u'_B} = \frac{\sum_{n=1}^{\infty} \frac{1}{n^2}}{\sum_{n=1}^{\infty} \frac{1}{\left(n - \frac{1}{2}\right)^2}} = \frac{1}{3} \tag{5}$$

Thom has shown in reference 2 that qg for a single line source may be replaced by $\sum qg$, the distribution of sources and sinks representing the wing section. Therefore, since the $\sum qg$ terms would also cancel each other, the result obtained for a single line source in equation (5) is identical to that for a complete wing.

Body of Revolution

A body of revolution may be represented by a distribution of point sources and sinks along the tunnel center line. The axial velocity increment due to a single image point source (fig. 2) is

$$u_1 = \frac{q'}{4\pi r^2} \sin\theta = \frac{q'g}{4\pi r^3} \tag{6}$$

at point A in the center of the basic tunnel, ${r_A}^2 = g^2 + n^2H^2 + m^2W^2$. Then, substituting into equation (6),

$$u_{1_A} = \frac{q'g}{4\pi W^3} \frac{1}{\left(\frac{g^2}{W^2} + n^2 \frac{H^2}{W^2} + m^2\right)^{3/2}} \tag{7}$$

Again omitting the central source and summing for the remainder with the assumption that g^2/W^2 is negligible compared with $n^2 \frac{H^2}{W^2}$ and m^2 gives

$$u_A = \frac{q'g}{4\pi W^3}\left[4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left(\frac{1}{n^2\frac{H^2}{W^2} + m^2}\right)^{3/2} + 2\sum_{\substack{n=1\\(m=0)}}^{\infty}\left(\frac{1}{n^2\frac{H^2}{W^2}}\right)^{3/2} + 2\sum_{\substack{m=1\\(n=0)}}^{\infty}\left(\frac{1}{m^2}\right)^{3/2}\right] \tag{8}$$

Similarly, it can be shown that the wall velocities u'_B and u'_C are

$$u'_B = \frac{q'g}{4\pi W^3}\left\{4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{1}{\left(n - \frac{1}{2}\right)^2\frac{H^2}{W^2} + m^2}\right]^{3/2} + 2\sum_{\substack{n=1\\(m=0)}}^{\infty}\left[\frac{1}{\left(n - \frac{1}{2}\right)^2\frac{H^2}{W^2}}\right]^{3/2}\right\} \tag{9}$$

and

$$u'_C = \frac{q'g}{4\pi W^3}\left\{4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\left[\frac{1}{n^2\frac{H^2}{W^2} + \left(m - \frac{1}{2}\right)^2}\right]^{3/2} + 2\sum_{\substack{m=1\\(n=0)}}^{\infty}\left[\frac{1}{\left(m - \frac{1}{2}\right)^2}\right]^{3/2}\right\} \tag{10}$$

The latter two equations include the effect of the primary source, which must be omitted in the calculation of u_A.

Straight, Untapered, Finite-Span Wing

A finite-span wing may be represented by a distribution of finite-length sources and sinks. The axial velocity increment produced by a single source element is (fig. 3), for this case,

$$du_1 = \frac{q\ dy_1}{4\pi r^2}\sin\theta = \frac{qg\ dy_1}{4\pi r^3} \tag{11}$$

For the point A in the center of the basic tunnel, the general expression for the square of the distance from the source element is $r_A{}^2 = g^2 + n^2H^2 + \left(mW + y_1\right)^2$. Integration across the image span gives the total contribution at A of one image; namely,

$$u_{1_A} = \frac{qg}{4\pi}\int_{-s}^{s} \frac{dy_1}{\left[g^2 + n^2H^2 + \left(mW + y_1\right)^2\right]^{3/2}} \tag{12}$$

Performance of the integration leads to the result

$$u_{1_A} = \frac{qg}{4\pi\left(n^2H^2 + g^2\right)}\left\{\frac{mW + s}{\left[g^2 + n^2H^2 + (mW + s)^2\right]^{1/2}} - \frac{mW - s}{\left[g^2 + n^2H^2 + (mW - s)^2\right]^{1/2}}\right\} \tag{13}$$

which is the same as equation 16, reference 2. Making equation (13) nondimensional and again neglecting the $\frac{g^2}{W^2}$ terms give

$$u_{1_A} = \frac{qg}{4\pi W^2}\left(\frac{1}{n^2 \frac{H^2}{W^2}}\right)\left\{\frac{m + \frac{s}{W}}{\left[n^2 \frac{H^2}{W^2} + \left(m + \frac{s}{W}\right)^2\right]^{1/2}} - \frac{m - \frac{s}{W}}{\left[n^2 \frac{H^2}{W^2} + \left(m - \frac{s}{W}\right)^2\right]^{1/2}}\right\} \tag{14}$$

As before for the case of the body of revolution, the total velocity increment is

$$u_A = \left[4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty} + 2\sum_{\substack{m=1 \\ (n=0)}}^{\infty} + 2\sum_{\substack{n=1 \\ (m=0)}}^{\infty}\right]u_{1_A} \tag{15}$$

However, it will be noted that the single summation for which $n = 0$ leads only to the indeterminate quantity $\frac{0}{0}$. This may be evaluated by the application of L' Hospital's Rule, finally giving for a single source line

$$u_A = \frac{qg}{4\pi W^2}\left(4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{n^2\frac{H^2}{W^2}}\left\{\frac{m+\frac{s}{W}}{\left[n^2\frac{H^2}{W^2}+\left(m+\frac{s}{W}\right)^2\right]^{1/2}}-\frac{m-\frac{s}{W}}{\left[n^2\frac{H^2}{W^2}+\left(m-\frac{s}{W}\right)^2\right]^{1/2}}\right\}+\right.$$

$$\left.\sum_{\substack{m=1\\(n=0)}}^{\infty}\left[\frac{1}{\left(m-\frac{s}{W}\right)^2}-\frac{1}{\left(m+\frac{s}{W}\right)^2}\right]+2\sum_{\substack{n=1\\(m=0)}}^{\infty}\frac{1}{n^2\frac{H^2}{W^2}}\left[\frac{2\frac{s}{W}}{\left(n^2\frac{H^2}{W^2}+\frac{s^2}{W^2}\right)^{1/2}}\right]\right) \tag{16}$$

Similarly, the total axial velocity increments at points B and C on the walls of the tunnel are

$$u'_B = \frac{qg}{4\pi W^2}\left(4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{\left(n-\frac{1}{2}\right)^2\frac{H^2}{W^2}}\left\{\frac{m+\frac{s}{W}}{\left[\left(n-\frac{1}{2}\right)^2\frac{H^2}{W^2}+\left(m+\frac{s}{W}\right)^2\right]^{1/2}}-\right.\right.$$

$$\left.\frac{m-\frac{s}{W}}{\left[\left(n-\frac{1}{2}\right)^2\frac{H^2}{W^2}+\left(m-\frac{s}{W}\right)^2\right]^{1/2}}\right\}+2\sum_{\substack{n=1\\(m=0)}}^{\infty}\frac{1}{\left(n-\frac{1}{2}\right)^2\frac{H^2}{W^2}}\times$$

$$\left.\left\{\frac{2\frac{s}{W}}{\left[\left(n-\frac{1}{2}\right)^2\frac{H^2}{W^2}+\frac{s^2}{W^2}\right]^{1/2}}\right\}\right) \tag{17}$$

and

$$u'_{C} = \frac{qg}{4\pi W^2}\left[4\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{1}{n^2\frac{H^2}{W^2}}\left(\frac{\left(m-\frac{1}{2}\right)+\frac{s}{W}}{\left\{n^2\frac{H^2}{W^2}+\left[\left(m-\frac{1}{2}\right)+\frac{s}{W}\right]^2\right\}^{1/2}} - \frac{\left(m-\frac{1}{2}\right)-\frac{s}{W}}{\left\{n^2\frac{H^2}{W^2}+\left[\left(m-\frac{1}{2}\right)-\frac{s}{W}\right]^2\right\}^{1/2}}\right) + \sum_{\substack{m=1\\(n=0)}}^{\infty}\left\{\frac{1}{\left[\left(m-\frac{1}{2}\right)-\frac{s}{W}\right]^2} - \frac{1}{\left[\left(m-\frac{1}{2}\right)+\frac{s}{W}\right]^2}\right\}\right] \tag{18}$$

Furthermore, at any point y along the span of the wing, the result is

$$u_y = \frac{qg}{4\pi W^2}\left[2\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty}\frac{1}{n^2\frac{H^2}{W^2}}\left(\frac{\left(m-\frac{y}{W}\right)+\frac{s}{W}}{\left\{n^2\frac{H^2}{W^2}+\left[\left(m-\frac{y}{W}\right)+\frac{s}{W}\right]^2\right\}^{1/2}} - \frac{\left(m-\frac{y}{W}\right)-\frac{s}{W}}{\left\{n^2\frac{H^2}{W^2}+\left[\left(m-\frac{y}{W}\right)-\frac{s}{W}\right]^2\right\}^{1/2}}\right) + \frac{1}{2}\sum_{\substack{m=-\infty\\(\text{except } m=0)\\(n=0)}}^{\infty}\left\{\frac{1}{\left[\left(m-\frac{y}{W}\right)-\frac{s}{W}\right]^2} - \frac{1}{\left[\left(m-\frac{y}{W}\right)+\frac{s}{W}\right]^2}\right\}\right] \tag{19}$$

As before, the primary source line which corresponds to the wing is not included in calculating the velocity increments at any point in the wing.

Swept, Untapered, Finite-Span Wing

In the preceding treatment for unswept bodies the analyses were carried out for single line and point sources as a simplification for bodies which could be represented by a combination of line and point sources and sinks. Thus, for a given value of g, all the image sources are in the same plane and a constant distance from the plane of A, B, and C, namely, g. Obviously, this condition no longer holds for swept bodies in which g would be a function both of span and angle of sweep, and hence the simplified treatment is no longer applicable. It is then necessary to proceed to a more general representation of the flow field involving the use of both sources and sinks. One of the simplest of such combinations is the doublet and that is what is used.

The Stokes stream function for a point doublet is given by (reference 3)

$$\psi = -\mu' \frac{w^2}{r^3} \tag{20}$$

where $\mu' = \frac{2aq'}{4\pi}$. The point-doublet strength μ' remains finite as a, the distance of the point source and sink from the doublet origin, approaches zero and q' goes to infinity. The velocity in the x-direction caused by the doublet element, whose axis is parallel to the x-axis (fig. 4), is given by

$$v_x = -\frac{1}{w} \frac{\partial \psi}{\partial w} = \mu' \left(\frac{2 - 3 \frac{w^2}{r^2}}{r^3} \right) \tag{21}$$

Then, for a swept-wing element, with doublet-element axes parallel to the x-axis,

$$du_1 = \mu \left(\frac{2 - 3 \frac{{w_1}^2}{r^2}}{r^3} \right) dy_1 \tag{22}$$

The square of the distance from the doublet element to A is $r_A{}^2 = x_1{}^2 + n^2H^2 + \left(mW + y_1\right)^2$. Considering Λ_0 positive for sweepback, then for the right wing with respect to the model, $\frac{x_1}{y_1} = \tan\Lambda_0$, and for the left wing, $\frac{-x_1}{y_1} = \tan\Lambda_0$. Having eliminated x_1, the expression for the contribution of a single-image doublet is

$$u_{1_A} = \mu \int_{-s}^{s} \left\{ \frac{2}{\left[y_1{}^2\tan^2\Lambda_0 + n^2H^2 + \left(mW + y_1\right)^2\right]^{3/2}} - \frac{3\left[n^2H^2 + \left(mW + y_1\right)^2\right]}{\left[y_1{}^2\tan^2\Lambda_0 + n^2H^2 + \left(mW + y_1\right)^2\right]^{5/2}} \right\} dy_1 \tag{23}$$

It should be noted that a change of sign between the right and left halves of a wing makes no difference mathematically. Therefore, the swept wing acts in exactly the same manner as a wing yawed at an angle Λ_0. Performing the integration and letting

$$a_A{}' = n^2 \frac{H^2}{W^2} + m^2$$

$$b_A{}' = 2m$$

$$c = \tan^2\Lambda_0 + 1$$

$$d_A{}' = 4a_A{}'c - \left(b_A{}'\right)^2$$

$$s' = \frac{s}{W}$$

$$X_{A+}{}' = a_A{}' + b_A{}'s' + c(s')^2$$

$$X_{A-}{}' = a_A{}' - b_A{}'s' + c(s')^2$$

leads to the result

$$u_{1_A} = \frac{\mu}{W^2}\left(\frac{1}{\left(X_{A+}'\right)^{1/2}}\left\{\frac{\left(b_A' + 2cs'\right)}{d_A'}\left[4 + \left(\frac{1}{X_{A+}'} + \frac{8c}{d_A'}\right)\left(\frac{\left(b_A'\right)^2}{c} - 2a_A'\right) - \frac{2\left(4a_A'c + \left(b_A'\right)^2\right)}{cd_A'}\right] + \frac{b_A'}{cX_{A+}'}\left(1 - \frac{2a_A'}{d_A'}\right) - \frac{s'\left(2\left(b_A'\right)^2 - 4a_A'c\right)}{cd_A'X_{A+}'}\right\} - \frac{1}{\left(X_{A-}'\right)^{1/2}}\left\{\frac{\left(b_A' - 2cs'\right)}{d_A'}\left[4 + \left(\frac{1}{X_{A-}'} + \frac{8c}{d_A'}\right)\left(\frac{\left(b_A'\right)^2}{c} - 2a_A'\right) - \frac{2\left(4a_A'c + \left(b_A'\right)^2\right)}{cd_A'}\right] + \frac{b_A'}{cX_{A-}'}\left(1 - \frac{2a_A'}{d_A'}\right) + \frac{s'\left(2\left(b_A'\right)^2 - 4a_A'c\right)}{cd_A'X_{A-}'}\right\}\right) \tag{24}$$

The total velocity increment at A may be obtained by the same summation as indicated for the straight wing in equation (15). However, for m and n equal to zero, the constants given above become:

For $m = 0$,

$$a_A' = n^2\frac{H^2}{W^2}$$

$$b_A' = 0$$

$$d_A' = 4a_A'c$$

$$X_{A+} = X_{A-}' = a_A' + c(s')^2 = X_A'$$

and, for $n = 0$,

$$a_A' = m^2$$

The final result is then

$$u_A \frac{W^2}{\mu} = \left(4 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} + 2 \sum_{\substack{m=1 \\ (n=0)}}^{\infty}\right)\left(\frac{1}{\left(X_{A+}'\right)^{1/2}}\left\{\frac{\left(b_A' + 2cs'\right)}{d_A'}\left[4 + \left(\frac{1}{X_{A+}'} + \frac{8c}{d_A'}\right)\left(\frac{\left(b_A'\right)^2}{c} - 2a_A'\right) - \frac{2\left(4a_A'c + \left(b_A'\right)^2\right)}{cd_A'}\right] + \frac{b_A'}{cX_{A+}'}\left(1 - \frac{2a_A'}{d_A'}\right) - \frac{s'\left(2\left(b_A'\right)^2 - 4a_A'c\right)}{cd_A'X_{A+}'}\right\} - \frac{1}{\left(X_{A-}'\right)^{1/2}}\left\{\frac{\left(b_A' - 2cs'\right)}{d_A'}\left[4 + \left(\frac{1}{X_{A-}'} + \frac{8c}{d_A'}\right)\left(\frac{\left(b_A'\right)^2}{c} - 2a_A'\right) - \frac{2\left(4a_A'c + \left(b_A'\right)^2\right)}{cd_A'}\right] + \frac{b_A'}{cX_{A-}'}\left(1 - \frac{2a_A'}{d_A'}\right) + \frac{s'\left(2\left(b_A'\right)^2 - 4a_A'c\right)}{cd_A'X_{A-}'}\right\}\right) + 2 \sum_{\substack{n=1 \\ (m=0)}}^{\infty}\left\{\frac{1}{\left(X_A'\right)^{1/2}}\left[-\frac{2s'}{a_A'}\left(\frac{a_A'}{X_A'} + \frac{1}{c}\right) + \frac{2s'}{cX_A'}\right]\right\} \tag{25}$$

In following a similar procecure to obtain u'_B and u'_C, it is found that the constants are the same as for u_A except that the subscripts are changed to B and C, respectively, and $n \longrightarrow n - \frac{1}{2}$ at B and $m \longrightarrow m - \frac{1}{2}$ at C. The summations are the same as for the straight wing; namely,

$$u'_B \frac{W^2}{\mu} = 4 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left(\frac{1}{\left(X_{B+}'\right)^{1/2}} \left\{ \frac{\left(b_B' + 2cs'\right)}{d_B'} \left[4 + \left(\frac{1}{X_{B+}'} + \frac{8c}{d_B'} \right) \left(\frac{\left(b_B'\right)^2}{c} - 2a_B' \right) - \right.\right.\right.$$

$$\left.\left. \frac{2\left(4a_B' + \left(b_B'\right)^2\right)}{cd_B'} \right] + \frac{b_B'}{cX_{B+}'} \left(1 - \frac{2a_B'}{d_B'} \right) - \frac{s'\left(2\left(b_B'\right)^2 - 4a_B'c\right)}{cd_B'X_{B+}'} \right\} -$$

$$\frac{1}{\left(X_{B-}'\right)^{1/2}} \left\{ \frac{\left(b_B' - 2cs'\right)}{d_B'} \left[4 + \left(\frac{1}{X_{B-}'} + \frac{8c}{d_B'} \right) \left(\frac{\left(b_B'\right)^2}{c} - 2a_B' \right) - \right.\right.$$

$$\left.\left.\left. \frac{2\left(4a_B'c + \left(b_B'\right)^2\right)}{cd_B'} \right] + \frac{b_B'}{cX_{B-}'} \left(1 - \frac{2a_B'}{d_B'} \right) + \frac{s'\left(2\left(b_B'\right)^2 - 4a_B'c\right)}{cd_B'X_{B-}'} \right\} \right) +$$

$$2 \sum_{\substack{n=1 \\ (m=0)}}^{\infty} \left\{ \frac{1}{\left(X_B'\right)^{1/2}} \left[-\frac{2s'}{a_B'} \left(\frac{a_B'}{X_B'} + \frac{1}{c} \right) + \frac{2s'}{cX_B'} \right] \right\} \tag{26}$$

and

$$u'_C \frac{W^2}{\mu} = \left(4 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} + 2 \sum_{\substack{m=1 \\ (n=0)}}^{\infty}\right)\left(\frac{1}{\left(X_{C+}'\right)^{1/2}}\left\{\frac{\left(b_C' + 2cs'\right)}{d_C'}\left[4 + \left(\frac{1}{X_{C+}'} + \frac{8c}{d_C'}\right)\left(\frac{\left(b_C'\right)^2}{c} - 2a_C'\right) - \frac{2\left(4a_C'c + \left(b_C'\right)^2\right)}{cd_C'}\right] + \frac{b_C'}{cX_{C+}'}\left(1 - \frac{2a_C'}{d_C'}\right) - \frac{s'\left(2\left(b_C'\right)^2 - 4a_C'c\right)}{cd_C'X_{C+}'}\right\} - \frac{1}{\left(X_{C-}'\right)^{1/2}}\left\{\frac{\left(b_C' - 2cs'\right)}{d_C'}\left[4 + \left(\frac{1}{X_{C-}'} + \frac{8c}{d_C'}\right)\left(\frac{\left(b_C'\right)^2}{c} - 2a_C'\right) - \frac{2\left(4a_C'c + \left(b_C'\right)^2\right)}{cd_C'}\right] + \frac{b_C'}{cX_{C-}'}\left(1 - \frac{2a_C'}{d_C'}\right) + \frac{s'\left(2\left(b_C'\right)^2 - 4a_C'c\right)}{cd_C'X_{C-}'}\right\}\right) \tag{27}$$

Support Struts

The method previously used to calculate the velocity ratios for aerodynamic bodies may also be applied to support struts. However, in order to avoid infinite velocity increments at the junctions of struts and model, it is necessary to consider the support system in the tunnel as an integral part of the model. With this viewpoint, velocity ratios could also be calculated for struts in the same manner as for wings, in which just the images are summed in order to calculate u_A. The span of the struts would be constant at $2s/W = 1.0$ in a reoriented tunnel for which $H/W = \sqrt{2}$.

Using points A and C, where C would now be at the top of the reoriented tunnel, the velocity ratio u_A/u'_C for half wings or struts would be identical to those for complete wings or double struts, both completely spanning the tunnel. There is one simple case for which the result is immediately known, namely, a single, centrally mounted, unswept strut for which the two-dimensional result of 1/3 applies (see section Two-Dimensional Wing). In general, if other than a single-strut support system were used, the velocity ratio would be a function of the strut spacing used as well as of the angle of sweep. The problem would be further complicated by the presence of a rear strut, which is frequently the case.

Because of the additional complexity involved and the expected difficulty in separating the total wall velocity increment into the separate effects due to the solid blocking of the support system, model wing, and model fuselage and to the wake blocking of each component, all of which may have different velocity ratios, no general solution of the support-strut case has been presented. It would be simpler and probably more accurate to perform a complete calibration of the area in which a model would normally be mounted, with the struts installed. During this calibration, the wall pressures at B and C could also be obtained, thus giving base values which include both the solid and wake blocking and interference effects of the model support system.

NUMERICAL CALCULATIONS AND RESULTS

The methods used in summing the doubly infinite series are explained in detail in appendix A. Briefly, calculations are made for each image up to a finite number $n = m = n_1$. The remainders are obtained by direct integration from n_1 to infinity after making certain simplifying assumptions. Unfortunately, the series convergence is not very rapid and it is necessary to take n_1 as high as seven in most cases. Fortunately, however, there is a negligible difference between the remainder terms for the swept and straight wings, since the effect of sweep rapidly diminishes as the distance between images and tunnel increases. Hence the very difficult problem of attempting to integrate the complicated remainders for the swept wings is avoided. In all

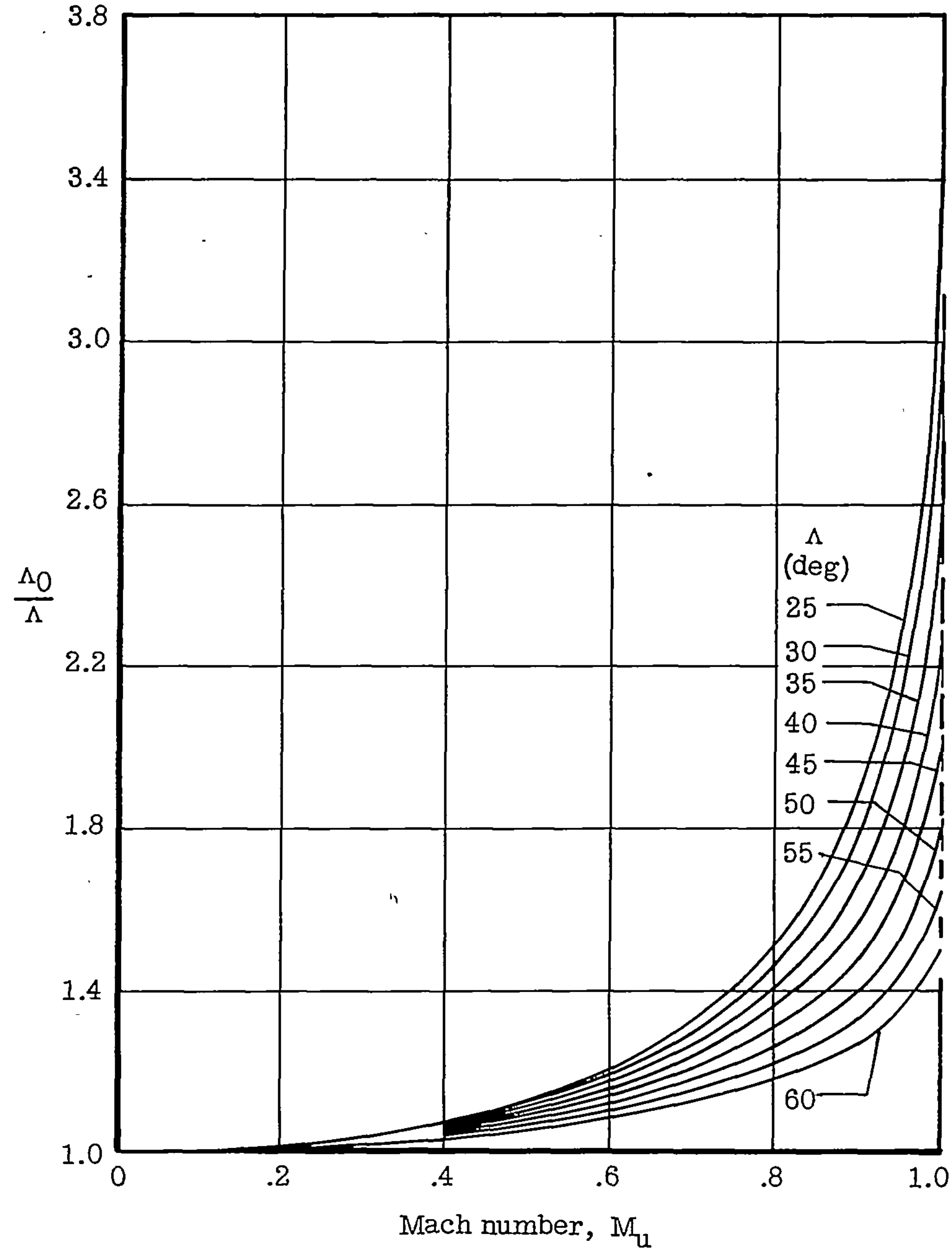

Figure 11.- Variation of Λ_0/Λ with Mach number.

$$\tan \Lambda_0 = \frac{\tan \Lambda}{\sqrt{1 - M_u^2}}.$$

NACA-Langley - 6-19-51 - 1000